

Exam I, MTH 213, Spring 2019

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SCORE = $\frac{57}{58}$ (□ (MW) ☑ (UTR))

QUESTION 1. (3 points) Use truth table and convince me that $\overline{(a+b)} + c = (c+\overline{a})(c+\overline{b})$

a	b	c	$\bar{a}$	$\bar{b}$	$\overline{(a+b)}$	$\overline{(a+b)}+c$	$(c+\bar{a})$	$(c+\bar{b})$	$(c+\bar{a})(c+\bar{b})$
0	0	0	1	1	0 / 1	1	1	1	1
0	0	1	1	1	0 / 1	1	1	1	1
0	1	0	1	0	1 / 0	0	1	0	0
0	1	1	1	0	1 / 0	1	1	1	1
1	0	0	0	1	1 / 0	0	0	1	0
1	0	1	0	1	1 / 0	1	1	1	1
1	1	0	0	0	1 / 0	0	0	0	0
1	1	1	0	0	1 / 0	1	1	1	1

They are identical

QUESTION 2. (3 points) (SHOW THE STEPS) Find $(413)_8 \times (61)_8$

$$\begin{array}{r} (413)_8 \\ \times\ (61)_8 \\ \hline 413 \\ 31620 \\ \hline (31433)_8 \end{array}$$

$\Rightarrow (31433)_8$

QUESTION 3. (3 points) (SHOW THE STEPS) Convert 5815 to base 16.

$(5815)_{10} \Rightarrow 16$

$5815 \div 16$

Q	R
363	7
22	11 ← B
1	6
0	1

$(5815)_{10} \Rightarrow (16B7)_{16}$

QUESTION 4. (6 points) (SHOW STEPS) Let $d = gcd(366, 237)$. Find d, then find a, b such that $d = 366a + 237b$.

$366 = 237 * 1 + 129$

$237 = 129 * 1 + 108$

$129 = 108 * 1 + 21$

$108 = 21 * 5 + 3$

$21 = 3 * 7 + 0$

$d = gcd(366, 237) = 3$

$\boxed{d = 3}$

$3 = 108 - (21 * 5)$

$3 = 108 - (129 - (108 \times 1))5$

$3 = 108 - 129 \times 5 + 108 \times 5$

$3 = 108 \times 6 - 129 \times 5$

$3 = (237 - (129 \times 1))6 - 129 \times 5$

$3 = 237 \times 6 - 129 \times 6 - 129 \times 5$

$3 = 237 \times 6 - 129 \times 11$

$3 = 237 \times 6 - (366 - (237 \times 1)) \times 11$

$3 = 237 \times 6 - 366 \times 11 + 237 \times 11$

$3 = 237 \times 17 - 366 \times 11$

$\boxed{a = -11 \quad b = 17}$

$d = 366(-11) + 237(17) = 3$

QUESTION 5. (7 points) (SHOW STEPS) Let X be number of laptops in a store. Given $2970 < X < 3960$, $X \equiv 2 \ (mod\ 9)$, $X \equiv 7 \ (mod\ 10)$, and $X \equiv 10 \ (mod\ 11)$. Find the value of X.

$x \bmod 9 = 2$
$x \bmod 10 = 7$
$x \bmod 11 = 10$

$n = 9 \times 10 \times 11 = 990$

$m_1 = 110 \quad m_1^{-1} = 5$
$m_2 = 99 \quad m_2^{-1} = 9$
$m_3 = 90 \quad m_3^{-1} = 6$

$(2 \times 110 \times 5) + (7 \times 99 \times 9) + (10 \times 90 \times 6) = 1100 + 6237 + 5400 = 12737$

$X = 12737 \bmod 990 = 857$

$857 + (990k) \quad k \in \mathbb{Z}$

$857 + (990 \times 3) = 3827$

QUESTION 6. (6 points) Write down T or F

(i) $\exists x \in R$ such that $x^2 + 4 = 20$ if and only if $x + 5 = 1$. ~~T~~ (F)

(ii) $\exists!\, x \in Q^*$ such that $\forall y \in Z$, we have $4x^2y - y = 0$ F

(iii) $\forall x \in Z, \exists! y \in Q^*$ such that $yx - 3x = 0$ F

(iv) $\forall x \in R^*, \exists! y \in Z$ such that $y^3x + 8x = 0$ T

(v) If $x^3 = -8x$ for some $x \in N$, then $x + 32 = 30$ ~~T~~ (F)

(vi) If $x^2 - 3 = 0$ for some $x \in Q$ or $y^2 - 5 = 0$ for some $y \in R$, then $2x^2 = 6$ and $3y^2 = 15$ ~~T~~ (F)

$y(4x^2 - 1) = 0$
$x(y - 3) = 0$
$x(y^3 + 8) = 0$

$\sqrt{3}, -\sqrt{3}$ F $\sqrt{5}, -\sqrt{5}$

-3

QUESTION 7. (Show steps)

(i) **(3 points)** What is $7^{211} \ (mod\ 15)$?

$m = 15 = 3^1 \times 5^1$

$\phi(m) = (3-1).3^0.(5-1).5^0 = 8$

$\frac{211}{8} = 26\frac{3}{8} \Rightarrow \underbrace{7^{26.8}}_{=1} . 7^3 \ (\text{mod } 15)$

$7^3 (\text{mod } 15) = \underline{13}$

(ii) **(4 points)** Find all possible values of X over Planet Z_{21} where $9X = 15$.

$9x \bmod 21 = 15$

$\gcd(9, 21) = 3$

$3 | 15 = \text{yes}$

$x_1 = 4 \quad x_2 = 11 \quad x_3 = 18$

$d = \frac{n}{\gcd} = \frac{21}{3} = 7$

(iii) **(2 points)** Find all possible values of X over Planet Z where $9X \equiv 15 \ (mod\ 21)$.

$d = \frac{n}{\gcd} = \frac{21}{3} = 7$

$\{4 + 7k, \ k \in \mathbb{Z}\}$

(iv) **(3 points)** Let $n = (125)(81) = 10125$. How many positive integers < 10125 such that $gcd(each\ integer, 10125) = 3$.

$10125 \div 3 = 3375$

$81 \div 3 = 27$

$\Rightarrow 125 . 27 = 3375$

$m = 5^3 . 3^3$

$\phi(m) = (5-1).5^2.(3-1).3^2 = 4.5^2.2.3^2 = \underline{1800}$

1800 integers

QUESTION 8. **(8 points)** Let $A = \{3, \{4\}, \{3\}, 2, \{2\}, 13, -2\}$, $B = \{2, \{4\}, 5, \{2\}, 8\}$, and $U = \{3, \{3\}, 2, \{2\}, \{4\}, 5, 8, \{7\}, 10, -2, 13\}$ (be the universal set). Then

(i) Find $B - A$.

$B - A = \{5, 8\}$ ✓

(ii) Find $\overline{B} \cap A$

$\overline{B} = \{3, \{3\}, \{7\}, 10, -2, 13\}$ $\Rightarrow$ $\overline{B} \cap A = \{3, \{3\}, -2, 13\}$ ✓

(iii) True of F

- $\{\{3\}, \{13\}\} \subseteq P(A)$ F ✗ (T)
- $\{2, \{2\}\} \subseteq B$. T ✓
- $\{\{4\}, \{8\}\} \in P(B)$ F ✓
- $\{3\} \in A - B$ T ✓

QUESTION 9. **(6 points))** Consider the following code

For $k = 4$ to $(6n^2 + 3)$
$S = k^4 + 3 * k^2 + K + 4$ $\Rightarrow 8$
For $i = 2$ to $(6k + 6)$
$L = 7 * i^6 + 3 * i^2 + 10$ $\Rightarrow 10$
next i
next k

(i) Find the exact number of addition, subtraction, multiplication that the code executed.

Outer loop: # of terms $= 6n^2 + 3 - 4 + 1 = \underline{6n^2}$

# of operations $= 8$

total in outer $= 8(6n^2) = 48n^2$

Inner loop: # of terms $= 6K + 6 - 2 + 1 = \underline{6K+5}$

# of operations $= 10$

first loop: $(6(4)+5)10 = 290$

last loop: $(6(6n^2+3)+5)10 = (36n^2 + 23)10 = 360n^2 + 230$

total # of operations $= 48n^2 + \dfrac{[290 + (360n^2 + 230)] \times 6n^2}{2}$ ✓

(ii) Find the complexity of the code.

$O[\text{code}] = n^4$, degree of 4 ✓

QUESTION 10. **(4 points))** (Show the steps) Given $a_n = 10a_{n-1} - 25a_{n-2}$ such that $a_0 = 3$, and $a_1 = 45$. Find a general formula for a_n, then find a_6.

$a_n = 10a_{n-1} - 25a_{n-2}$

$x^n = 10x^{n-1} - 25x^{n-2}$

$\div x^{n-2}$

$x^2 = 10x - 25$

$x^2 - 10 + 25 = 0$

$(x-5)(x-5)$

$x_1 = 5 \quad x_2 = 5$

$a_n = c_1 x_1^n + c_2 n x_2^n$

$a_0 = c_1 5^0 + c_2 \cdot 0 \cdot 5^0 = \boxed{c_1 = 3}$ ✓

$a_1 = c_1 5^1 + c_2 \cdot 1 \cdot 5^1 = 5c_1 + 5c_2 = 45$ $\boxed{c_2 = 6}$

$a_n = 3 \cdot (5)^n + 6 \cdot n \cdot (5)^n$

$a_6 = 3 \cdot (5)^6 + 6 \cdot 6 \cdot (5)^6 = 609375$ ✓

Faculty information

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